Finite Element Analysis of
Magnetically Shielded Coils Using Homogenization Method

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The magnetically shielded wire, composed of a conducting wire coated by magnetic layers, can effectively reduce the eddy current losses due to the proximity effect. The conventional finite element method (FEM) needs unacceptably long computational time to analyze eddy currents in the multi-turn magnetically shielded coils. This paper proposes a homogenization method which models the coil region as a uniform material with complex permeability. The coil impedance evaluated by the homogenization-based FEM with coarse elements is shown to agree well with that obtained by the conventional FEM with fine elements.

Index Terms—**Magnetically shielded wire, skin effect, proximity effect, homogenization, complex permeability**

I. INTRODUCTION

Magnetically shielded wires are composed of a wire conductor and layered magnetic materials [1]. Increasing in the driving frequency of electric machines leads to increase in the eddy current losses in the wire due to the skin and proximity effects. When using magnetically shielded wires, the magnetic fluxes tends to pass not through the central conductor but through the magnetic thin layers. For this reason, the eddy current losses due to the proximity effect can effectively be reduced [2, 3].

In order to compute the eddy current losses in magnetically shielded wires by the conventional FE analysis, fairly fine spatial discretization is required so that the element size is sufficiently smaller than the skin depth. Moreover, air-gap and insulated film must also be subdivide into fine elements.

The authors have proposed a homogenization method so as to circumvent this problem [4, 5]. In this method, the coil is modeled as a uniform material with the homogenized complex permeability $\langle \dot{\mu} \rangle$ to consider the magnetic dipole fields caused by proximity effect. This method has been shown effective and accurate in the analysis of multi-turn coils [4] and magnetically shielded wire which has single non-conducting magnetic layer [5]. In this work, we extend these methods to homogenize a magnetically shielded wire which has conducting magnetic multi-layers.

II. FORMULATION

Let us consider a round magnetically shielded wire immersed in a uniform time-harmonic magnetic field $B_0 e^{j\omega t}$ perpendicular to the wire axis as show in Fig. 1. The complex permeability of a round magnetically shielded wire is shown to be expressed by

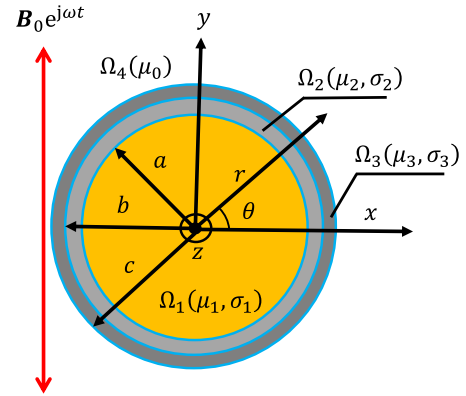$$\dot{\mu} = \mu_0 \frac{1 + \Psi(\omega)}{1 - \Psi(\omega)} \tag{1a}$$

Fig. 1 Cross-section of a magnetically shielded wire immersed in uniform, time-harmonic field. In this model, the magnetic layers are composed of conductive material.

where J_ν, N_ν denote the ν-th order Bessel and Neumann functions, respectively, $k_i = (1-j)/\delta_i$, ($\delta_i \cdots$ skin depth in Ω_i) and the definitions of coefficients $\zeta_i, \lambda_i, \tau_{ij}$ are given in TABLE I. The magnetically shielded coil region can be modeled as a uniform material with the macroscopic complex permeability which can be obtained by inserting $\dot{\mu}_r$ into the extended Ollendorff formula [4, 5]

TABLE I
DEFINITIONS OF COEFFICIENTS $\zeta_i, \lambda_i, \tau_{ij}$

λ_1	λ_2
$= \{-\tau_{12} J_1(k_1 a) J_1'(k_2 a)$ $+ J_1'(k_1 a) J_1(k_2 a)\} N_1(k_2 b)$ $+ \{-J_1'(k_1 a) N_1(k_2 a)$ $+ \tau_{12} J_1(k_1 a) N_1'(k_2 a)\} J_1(k_2 b)$	$= \{-\tau_{12} J_1(k_1 a) J_1'(k_2 a)$ $+ J_1'(k_1 a) J_1(k_2 a)\} N_1'(k_2 b)$ $+ \{-J_1'(k_1 a) N_1(k_2 a)$ $+ \tau_{12} J_1(k_1 a) N_1'(k_2 a)\} J_1'(k_2 b)$
$\zeta_1 = -\tau_{23} c^2 J_1'(k_3 b) N_1'(k_3 c)$ $\quad + \tau_{23} c^2 J_1'(k_3 c) N_1'(k_3 b)$	$\zeta_2 = -c^2 J_1(k_3 b) N_1'(k_3 c)$ $\quad + c^2 J_1'(k_3 c) N_1(k_3 b)$
$\zeta_3 = -\tau_{23} c N_1(k_3 c) J_1'(k_3 b)$ $\quad + \tau_{23} c J_1(k_3 c) N_1'(k_3 b)$	$\zeta_4 = -c J_1(k_3 b) N_1(k_3 c)$ $\quad + c J_1(k_3 c) N_1(k_3 b)$
$\tau_{12} = \delta_1 \mu_1 / \delta_2 \mu_2$	$\tau_{23} = \delta_2 \mu_2 / \delta_3 \mu_3$

$$\Psi(\omega) = \frac{-(\lambda_1 \zeta_1 - \lambda_2 \zeta_2) + (\lambda_1 \zeta_3 - \lambda_2 \zeta_4)\mu_3/k_3\mu_0}{\left[\begin{array}{l}\left(-\lambda_1 \tau_{23} J_1'(k_3 b) + \lambda_2 J_1(k_3 b)\right)\left(c N_1(k_3 c)\mu_3/k_3\mu_0 + c^2 N_1'(k_3 c)\right) \\ +\left(\lambda_1 \tau_{23} N_1'(k_3 b) - \lambda_2 N_1(k_3 b)\right)\left(c J_1(k_3 c)\mu_3/k_3\mu_0 + c^2 J_1'(k_3 c)\right)\end{array}\right]} \tag{1b}$$

$$\langle \dot{\mu}_r \rangle = 1 + \frac{\eta(\dot{\mu}_r - 1)}{1 + N(1 - \eta)(\dot{\mu}_r - 1)} \tag{2}$$

Moreover, it can be shown that the total power of a system including magnetically shielded coil can be obtained from

$$\frac{VI^*}{2} = \frac{j\omega}{2} \int_\Omega \mu |\boldsymbol{H}|^2 \mathrm{d}v + \kappa(\omega) \frac{\tilde{R}_0}{4} |I|^2 \tag{3}$$

where μ_r is set to $\langle \dot{\mu}_r \rangle$ in magnetically shielded coil, $\kappa(\omega)$ denotes the factor expressing the skin effect whose frequency characteristic is plotted in Fig. 2, $\tilde{R}_0 = l/(\pi c^2 \sigma_3)$, The first term in (3) includes the eddy current loss due to the proximity effect and time variation in the stored magnetic energy in air region Ω_4, while the second term represents the eddy current loss due to the skin effect. Magnetic vector potential $\boldsymbol{A}$ is computed by solving the equation

$$\mathrm{rot}\,(\nu\,\mathrm{rot}\,\boldsymbol{A}) = \boldsymbol{J} \tag{4}$$

where $\boldsymbol{J}$ is current density of the coil, the reluctance ν is set to $1/\langle \dot{\mu} \rangle$ in magnetically shielded coil region, and every conductivity of wire become zero because eddy currents are expressed by $\langle \dot{\mu}_r \rangle$. The impedance Z of magnetically shielded coil is calculated from $Z = 2P/|I|^2$.

III. NUMERICAL RESULTS

We compare Z computed by the proposed method and conventional FEM in which magnetically shielded coil is subdivided into fine elements. Because many-turn magnetically shielded coils cannot be analyzed by the latter method, we consider here the four-turn magnetically shielded coil shown in Fig. 3. The specification of the magnetically shielded coil is summarized in TABLE II. There are 958 and 89056 elements in the coil region for the former and latter analyses, respectively. Magnetic flux lines are shown in Fig. 4. In Fig. 5, Z is plotted against the normalized wire radius c/δ_3. Both real and imaginary components increase with frequency due to the skin and proximity effects. We conclude from Fig. 3, 4 that the proposed method has the same accuracy as the conventional FEM while the former solves much smaller FE equations.

IV. CONCLUSION

In this paper, a homogenization method to analyze eddy currents in magnetically shielded coil has been proposed. It has been shown that the impedance of magnetically shielded coil computed by the proposed method agree well with that computed by the conventional magnetically shielded coil.

In the long version, the three-dimensinoal analysis of wireless power transfer device including magnetically shielded coil using the proposed method will be reported.
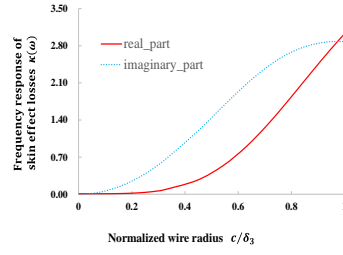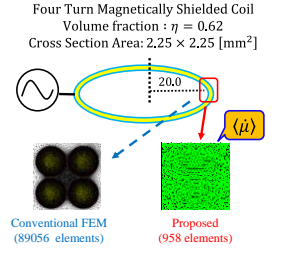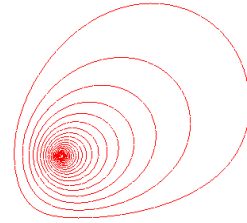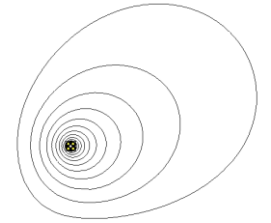

Fig. 2 Profile of $\kappa(\omega)$

Fig. 3 Four-turn magnetically shielded coil

TABLE II
SPECIFICATION OF MAGNETICALLY SHIELDED WIRE SHOWN IN FIG. 3

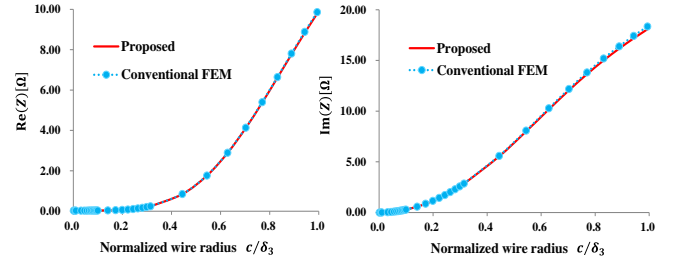|	length [mm]	relative permeability		conductivity [S/m]	
a	0.30	μ_{1r}	1	σ_1	5.76×10^7
b	0.40	μ_{2r}	100	σ_2	1.0×10^4
c	0.50	μ_{3r}	10	σ_3	1.0×10^5

(a) Proposed method　　　(b) Conventional FEM

Fig. 4 Magnetic flux lines distributions

(a) Real part　　　(b) Imaginary part

Fig. 5 Impedance of magnetically shielded coil shown in Fig. 4. The abscissa is the ratio of the wire radius c to the skin depth δ_3. In this model, dc-resistance of the coil is 30.8 [mΩ].

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